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Roll No. :

No. of Q. Paper : 143 I

Unique Paper Code : 42354302

Name of the Course : B.Sc.(Prog.)/ B.Sc.
Mathematical Sciences

Name of the Paper : Algebra

Semester : III

Time : 3 Hours

Maximum Marks : 75

Instructions for Candidates :

- (a) Write your Roll No. on the top immediately on receipt of this question paper.
- (b) Attempt any **Two** parts from each question.
- (c) **All** questions are compulsory.
- (d) **Marks** are indicated.

Unit-I

(a) Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix}; a \in \mathbb{R}; a \neq 0 \right\}$

Show that G is a group under matrix multiplication.

6

P.T.O.

- (b) (i) Let G be a group such that if a, b, c and $ab = ca \Rightarrow b = c$, then prove that G is abelian.
- (ii) Let $H = \{ x \in \mathbb{U}(20) : x \equiv 1 \pmod{3} \}$.
List all elements of H .
Prove or disprove that H is a subgroup of $\mathbb{U}(20)$.
- (c) Prove that the intersection of two subgroups of a group is a subgroup but their union is not.
2. (a) Define cyclic group. Prove that every cyclic group is Abelian. Is the converse true? Justify.
- (b) Give an example of a non cyclic group all whose proper subgroups are cyclic.
- (c) Let $\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 3 & 5 & 4 & 7 & 6 & 8 \end{bmatrix}$
and $\beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 3 & 8 & 7 & 6 & 5 & 2 & 4 \end{bmatrix}$
- (i) Write α and β as product of disjoint cycles.
- (ii) Find $o(\alpha\beta)$ and $o(\alpha^{-1})$
3. (a) Let 'a' be an element of a finite group G .
Prove that $a^{o(G)} = e$.

- (b) Consider the subgroup $H = \{1, 9\}$ of group $G = U(20)$ under multiplication modulo 20. Find the number of cosets of H in G and determine all the distinct cosets of H in G . 6

- (c) Prove that the center $Z(G)$ of a group G is a normal subgroup of G . 6

Unit- II

- (a) Prove that a non empty subset S of a ring R is a subring of R if and only if $a-b \in S$ and $ab \in S \forall a, b \in S$. 6.5

- (b) Prove that $\mathbb{Q}[\sqrt{2}] = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ is an integral domain. 6.5

- (c) (i) Let $\mathbb{Z}$ be the ring of integers and n be a fixed integer.

Show that $I = \langle n \rangle = \{nx : x \in \mathbb{Z}\}$ is an ideal of $\mathbb{Z}$. 3.5

- (ii) Give an example of a finite , non commutative ring . 3

Unit- III

- (a) Determine whether or not the set

$$\left\{ \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}$$

is linearly independent over $\mathbb{Z}_5$. 6.5

- (b) Define the linear span of a subset of a vector space $V(F)$ and prove that the linear span of a set S is a subspace of $V(F)$ containing S .
- (c) Determine whether or not $\{(1, 3, 2), (2, 0, 1), (1, 1, 1)\}$ form a basis of R^3 .

6. (a) Matrix of a linear transformation T with respect to basis $\{(1, 2), (0, 1)\}$ of R^2 is given

$$\text{by } \begin{bmatrix} 2 & 1 \\ -3 & -2 \end{bmatrix}.$$

Determine the linear transformation T .

- (b) Let U and V be two finite dimensional vector spaces over F . Let T from U to V be a linear transformation. If $\{u_1, u_2, u_3, \dots, u_n\}$ generates U then show that Range space of T is generated by $\{T(u_1), T(u_2), T(u_3), \dots, T(u_n)\}$.

- (c) Find the range, rank, kernel (Null space) and nullity of T where linear transformation $T: R^2 \rightarrow R^3$ is defined by $T(x, y) = (y, x + 2y, x + y)$.